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Welfare Comparisons under Vertical and Horizontal Mergers with Directly-Channelled Digital Goods

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Abstract

This paper analyzes how the emergence of directly-channelled digital goods affects the prices and welfare ranking among cases involving no integration, vertical mergers, and horizontal mergers. We consider an economy of digital-physical mashups where one upstream firm sells a product to two downstream distributors and also sells a directly-channelled electronic version of its product to consumers directly. Competition between direct and indirect distribution channels allows consumers to purchase either a physical good from the downstream firms or an electronic version of that good from the directly-channelled upstream firm. We find that there exists a possibility of reversed welfare ranking, in contrast to the traditional wisdom. The important policy implication is that when the digital and physical goods are strong substitutes, horizontal integration is welfare improving, because the presence of digitalization reduces the effects of double marginalization and market concentration, and decreases physical and digital prices. When relative bargaining power is considered, vertical integration may be the worst case.

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Moreover, an outside provider of the digital good is also discussed. In all scenarios, the digital good is likely to be cheaper than the physical good. The implications for antitrust policies are also provided. First, since our results reveal the possibility that a horizontal merger between downstream firms can be socially beneficial when the substitutability between the physical and digital version of the product is large, the antitrust authorities may take into account the situation of substitutability between physical and digital versions as one of the key factors for merger policies in different cases. Another policy implication is associated with vertical integration between the upstream firm and some of the downstream firms. The analytical results reveal that this type of vertical merger is welfare improving due to the effects of reducing double marginalization. Our results suggest examining whether the double marginalization is reduced significantly from vertical mergers in view of the role played by directly-channelled electronic versions of products.

Keywords: Vertical Merger, Horizontal Merger, Digital Goods, Social Welfare, Double Marginalization.

1. Introduction

Digital goods have grown rapidly in the past two decades, especially in music, books, videos, and software. The emergence of the Internet has led to digitalization of various physical goods and direct-channel sales. For instance, some publishers such as Macmillan sell electronic books directly to readers, instead of selling through online retailers such as Amazon. Over-the-top (OTT) content providers such as Netflix have defeated traditional video rentals such as Blockbuster, and Netflix now has more American subscribers than cable TV. Moreover, hotel rooms, air tickets, and software are sold via traditional retailers and official websites directly. Antitrust concerns in the digital economy have received much attention from the public, academia, and regulators, such as the U.S. Federal Trade Commission. Amidst those concerns, this paper attempts to introduce a directly-channelled digital good into merger issues, including vertical integration and horizontal integration, which has seldom been fully analyzed. We examine a novel question of how the welfare ranking and prices may change and be different from those in traditional studies. We also attempt to examine the merger policy implications in the economy of digital-physical mashups, and to discuss whether the current merger policy is in need of modification in this digital-physical economy. This study focuses on a digital-physical economy where one upstream firm either sells a product through two indirectly-channelled downstream distributors or sells the digital version of its product through a direct channel to consumers[1].

Traditionally, vertical and horizontal integration may yield efficiency gains but cause anticompetitive concerns. Efficiency gains could come from the reduction of transaction costs, economies of scale, and avoiding double marginalization. As for anticompetitive concerns, vertical integration is embedded with the incentive to foreclose competition, and horizontal integration leads to an increase in concentration.

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[1] For instance, traditionally, HBO sells its programs through cable TV indirectly to subscribers. Now HBO provides a direct-channeled HBO GO to consumers with an additional streaming option.

Related studies are as follows. Spengler (1950) suggested that vertical integration helps to avoid double marginalization[2], while Williamson (1971), Arrow (1975), and Aoki (1986) pointed out that vertical integration may either enhance performance or reduce uncertainties, and thus firms can obtain efficiency gains[3]. Hamilton and Mqasqas (1996) analyzed a model with conjecture variation and multiple firms, and demonstrated that vertical integration is in general socially desirable[4]. Chen (2001) further discussed the roles of competition in the upstream market and product differentiation in the downstream market[5]. Milliou and Petrakis (2007) analyzed the merger decision and the selection of contract types for upstream stores in a vertically related industry with a bargaining process[6]. Perry and Porter (1985) mentioned that the incentive of horizontal mergers depends on the conjecture variation among oligopoly firms and cost parameters[7]. Davidson and Mukherjee (2007) further considered the influence of free entry on horizontal mergers[8]. Recently, Zanchettin and Mukherjee (2017) considered product differentiation and emphasized how competition intensity in the downstream market affects the surplus from vertical integration[9]. Other discussion can be found in_Deneckere and Davidson (1985), Salinger (1988), Tirole (1988), Farrell and Shapiro (1990), Hart et

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2 Joseph Spengler, "Vertical Integration and Antitrust Policy," *58(4) Journal of Political Economy*, 347-352 (1950).

3 Oliver Williamson, "The Vertical Integration of Production: Market Failure Considerations," *61(1) American Economic Review*, 112-123 (1971); Kenneth Arrow, "Vertical Integration and Communication," *6(1) Bell Journal of Economics*, 173-183 (1975); Masahiko Aoki, "Horizontal vs. Vertical Information Structure of the Firm," *76(5) American Economic Review*, 971-983 (1986).

4 James Hamilton & Ibrahim Mqasqas, "Double Marginalization and Vertical Integration: New Lessons from Extensions of the Classic Case," *62(3) Southern Economics Journal*, 567-584 (1996).

5 Yongmin Chen, "On Vertical Mergers and Their Competitive Effects," *32(4) RAND Journal of Economics*, 667-685 (2001).

6 Chrysovalantou Milliou & Emmanuel Petrakis, "Upstream Horizontal Mergers, Vertical Contracts, and Bargaining," *25(5) International Journal of Industrial Organization*, 963-987 (2007).

7 Martin Perry & Robert Porter, "Oligopoly and the Incentive for Horizontal Merger," *75(1) American Economic Review*, 219-227 (1985).

8 Carl Davidson & Arijit Mukherjee, "Horizontal Mergers with Free Entry," *25(1) International Journal of Industrial Organization*, 157-172 (2007).

9 Piercarlo Zanchettin & Arijit Mukherjee, "Vertical Integration and Product Differentiation," *55 International Journal of Industrial Organization*, 25-57 (2017).

al. (1990), Ordover et al. (1990), Choi and Yi (2000), Mason and Phillips (2000), Chemla (2003) and Spector (2003)[10].

Antitrust concern in digital economics has received increased attention recently. Among numerous studies, Cabral (2020) warned firms against abusing their dominant position and suggested tightening consumer protection because of the disencouragement effect on innovation[11]. Rizzo (2020) studied the venture capital industry and pointed out digital incumbents may be dominant and the antitrust authorities should take better consideration about the ambiguous enforcement effect on *ex ante* incentives for innovation[12]. Economides and Lianos (2020) further examined the associations between privacy and antitrust in digital platforms such as Google and Facebook[13]. McGinnis and Sun (2020) analyzed the associated problems involved in dual antitrust enforcement through the Federal Trade Commission (FTC) and the Department of Justice (DOJ)[14]. Goldfarb and Tucker (2019) provided a comprehensive survey on the recent development

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10 Raymond Deneckere & Carl Davidson, "Incentives to Form Coalitions with Bertrand Competition," *16(4) Rand Journal of Economics*, 473-486 (1985); Michael Salinger, "Vertical Mergers and Market Foreclosure," *103(2) Quarterly Journal of Economic*, 345-356 (1988); Jean Tirole, *The Theory of Industrial Organization*, 1st ed., MIT Press (1988); Joseph Farrell & Carl Shapiro, "Horizontal Mergers: An Equilibrium Analysis," *80(1) American Economic Review*, 107-126 (1990); Oliver Hart, Jean Tirole, Dennis Carlton & Oliver Williamson, "Vertical Integration and Market Foreclosure," *Special Issue, Brookings Papers on Economic Activity*, 205-286 (1990); Janusz Ordover, Garth Saloner & Steven Salop, "Equilibrium Vertical Foreclosure," *80(1) American Economic Review*, 127-142 (1990); Jay Pil Choi & Sang-Seung Yi, "Vertical Foreclosure with the Choice of Input Specifications," *31(4) RAND Journal of Economics*, 717-743 (2000); Charles Mason & Owen Phillips, "Vertical Integration and Collusive Incentives: An Experimental Analysis," *18(3) International Journal of Industrial Organization*, 471-496 (2000); Gilles Chemla, "Downstream Competition, Foreclosure, and Vertical Integration," *12(2) Journal of Economics and Management Strategy*, 261-289 (2003); Spector, David, "Horizontal Mergers, Entry, and Efficiency Defences," *21(10) International Journal of Industrial Organization*, 1591-1600 (2003).

11 Luís Cabral, "Merger Policy in Digital Industries," *100866 Information Economics and Policy* (2020).

12 Andrea Minuto Razzo, "Digital Mergers: Evidence from the Venture Capital Industry Suggests That Antitrust Intervention Might Be Needed," *Ipaa051 Journal of European Competition Law & Practice*, 1-10 (2020).

13 Nicholas Economides & Ioannis Lianos, "Privacy and Antitrust in Digital Platforms," *21-01 NET Institute Working Paper*, 1-4 (2020).

14 John O. McGinnis & Linda Sun, "Unifying Antitrust Enforcement for the Digital Age," *20-20 Northwestern Public Law Research Paper*, 1-41 (2020).

of digital economics and examined whether and how digital technology change economic activity[15]. Specifically, they focused on five cost reductions in searching, replication, transportation, tracking, and verification, and analyzed the associated changes of economic activities. Among all the above literature, there is no formal analysis on physical-digital competition, which will be the main focus of this paper.

In a simple setting with a constant unit production cost and without information asymmetry, we emphasize the competition policy of demand-side competition, where a directly-channelled digital good is introduced as a substitute for the physical good[16]. The traditional welfare ranking of different market structures is that vertical integration is superior to non-integration, and horizontal integration is inferior to the above two structures. This is because vertical integration eliminates double marginalization and thus increases social welfare. In contrast, horizontal integration leads to a monopolistic downstream market and thus decreases social welfare. In our paper, with an electronic version of a product sold to consumers directly by an upstream firm, both the double marginalization effect and the market power effect are decreased. The reason is that the direct channel bears no burden of double marginalization, and a digital good implies a new competitor in the market. If these effects are strong enough, then the traditional welfare ranking will be changed. Specifically, the case of no integration may be the worst case when the physical good and the digital good are strong substitutes, because horizontal integration may result in lower physical and digital prices due to lower input prices. That is, horizontal mergers could be socially desirable even if there are no benefits from cost efficiency[17]. Since our framework does not consider the incentive to foreclose competition from potential entrants, vertical integration is always welfare enhancing, even though a

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[15] Avi Goldfarb & Catherine Tucker, "Digital Economics," *57(1) Journal of Economic Literature*, 3-43 (2019).

[16] Since our setting allows only one upstream firm, there is no substitutability of input supplies. Alternatively, the competition policy for the supply-side substitute has received attention from competition authorities, legal experts, and economists. For more discussion, refer to Franz Jürgen Säcker, *The Concept of the Relevant Product Market: Between Demand-side Substitutability in Competition Law*, 1. Aufl., Peter Lang GmbH (2008).

[17] If the average unit cost decreases when merges occur due to economies of scale, then horizontal integration may be much more socially desirable than in our framework.

digital good is introduced. Therefore, our welfare comparison will focus on the case of horizontal merger and the case of no integration. However, we still include the case of vertical integration as a reference for comparison. The price comparison of the electronic product in different market structures also depends on the substitutability between physical and digital goods. Moreover, digital goods are in general cheaper than physical goods under different market structures. Finally, our results can be extended to the case with bargaining power and the case with an outside digital provider.

The rest of this paper is organized as follows. Section 2 describes the model setting. Section 3 presents the benchmark case without any digital product. Section 4 demonstrates the welfare comparison with an electronic version of the product. Section 5 provides three extensions to include different bargaining power among firms, the case with an outside digital provider, and a rapidly growing digital market. Conclusions are offered in Section 6.

2. The Model Setting

Suppose there is a vertical structure of industry such that one upstream firm (U) sells a raw material (input) to two downstream firms (D_1 and D_2)[18]. The upstream firm also sells a directly-channelled electronic version of its product to consumers directly with a price p_e. Suppose there exists no vertical differentiation between the physical version and the digital version for consumers. That is, their intercepts of demand are identical. In the first stage, the upstream firm decides the input prices (w_1 and w_2) to the downstream firms, where one unit of input is produced to one unit of output with prices p_1 and p_2. In the second stage, the downstream firms and the upstream firm (which is also the provider of the digital good) engage in price competition and consumers make their decisions[19]. All

[18] Since our focus is welfare comparison instead of profit comparison as in most related literature, the structure assumes only one upstream firm in order to reduce the mathematical complexity.

[19] If we consider a structure such that there are two (or more) upstream firms (U_1 and U_2), then price competition in the upstream market induces marginal cost pricing in the input market. Therefore, symmetric vertical integration ($U_1 + D_1$ and $U_2 + D_2$) has no effect on the output prices. Moreover, horizontal integration in the downstream market leads to an increase in output prices, which is not socially desirable.

production costs are assumed to be zero, except the raw material sold to the downstream firms. The market structure is shown in Figure 1.

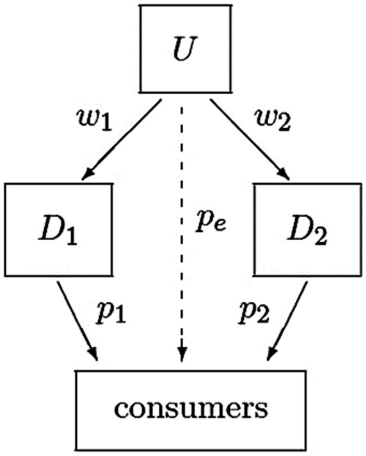

Figure 1. The structure of the market.

Source: Organized by the authors.

Suppose that the consumer surplus (CS) of a representative consumer who consumes q_1, q_2 and q_e is[20]

$$CS = \alpha(q_1 + q_2 + q_e) - \frac{1}{2}\left(\beta(q_1^2 + q_2^2 + q_e^2) + 2rq_1q_2 + 2\eta q_e(q_1 + q_2)\right) - p_1q_1$$

$$-p_2q_2 - p_eq_e.$$

The CS includes a quadratic utility in quantities minus expenditures, and implies the following inverse demand functions

$$p_i = \alpha - \beta Q_i - rQ_j - \eta Q_e, \quad i,j = 1,2, \quad i \neq j, \quad (1)$$

$$p_e = \alpha - \beta Q_e - \eta(Q_1 + Q_2), \quad (2)$$

[20] Our model considers a representative consumer. Alternatively, consider that consumers have different tastes for the choice between the physical and digital goods; say there exist a proportion of consumers who have no preference for the digital good, such as old-fashioned people. In this situation, the influence of digital goods will be reduced, and thus our results will become closer to the traditional results.

where p_i and Q_i are prices and quantities, α is the intercept, β is the slope, r and η are the measures of substitutability between firm 1 and firm 2, and between the physical product and the electronic product, respectively. The subscripts 1, 2 and e represent products of firms 1 and 2, and the electronic version, respectively. The demand function $Q_i(p_1, p_2, p_e)$, $i = 1,2,e$, can be expressed by solving (1) and (2) simultaneously. Let $\beta > 0$ and $r, \eta \in (0, \beta)$[21]. Assume η is not very close to β to avoid technical problems.

3. The Benchmark Case without Digital Goods

In this section, there are no digital goods, and the inverse demand becomes $p_1 = \alpha - \beta Q_1 - rQ_2$ and $p_2 = \alpha - \beta Q_2 - rQ_1$. The profit functions for the upstream firm and two downstream firms are

$$\pi_U = w_1 Q_1 + w_2 Q_2 = \frac{w_1(\alpha(\beta - r) - \beta p_1 + p_2 r)}{\beta^2 - r^2} + \frac{w_2(\alpha(\beta - r) - \beta p_2 + p_1 r)}{\beta^2 - r^2},$$

$$\pi_i = (p_i - w_i) \cdot Q_i = \frac{(p_i - w_i)(\alpha(\beta - r) - \beta p_i + p_j r)}{\beta^2 - r^2}, \quad i, j = 1,2, \quad i \neq j.$$

We can further discuss the production cost for the upstream firm, where the physical good has a positive marginal cost and the digital good has zero marginal cost. For instance, e-books or online videos can be duplicated at a negligible cost. With this extensive change in setting, our major results continue to hold[22].

3.1 No Integration

Without integration in the second stage, solving $\partial\pi_1/\partial p_1 = 0$ and $\partial\pi_2/\partial p_2 = 0$ for the downstream firms yields

$$p_i = \frac{\alpha(2\beta^2 - r^2 - \beta r) + 2\beta^2 w_i + \beta w_j r}{4\beta^2 - r^2}, \quad i, j = 1,2, \quad i \neq j.$$

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[21] Our framework can be extended to the case with negative values of r and η, which indicate that products may be complements. We do not discuss these cases for simplicity, since for practical purposes, direct-channel digital goods are substitutes for original physical goods.

[22] Further calculations reveal that the welfare ranking is not sensitive to this cost difference between the physical good and the digital good.

Plugging p_1 and p_2 into the profit function of the upstream firm π_U and solving $\partial\pi_U/\partial w_1 = 0$ and $\partial\pi_U/\partial w_2 = 0$ simultaneously yields $w_1^N = w_2^N = \frac{\alpha}{2}$, and $p_1^N = p_2^N = \frac{\alpha(3\beta-2r)}{2(2\beta-r)}$.

The social welfare is defined as $W^N = \pi_1 + \pi_2 + \pi_U + CS = \frac{\alpha^2\beta(7\beta-4r)}{4(r+\beta)(r-2\beta)^2}$, where the superscript N represents "no integration," which is the summation of profits for all firms and the consumer surplus.

3.2 Vertical Integration

Suppose the downstream firm 1 is merged by the upstream firm[23]. In the second stage, solving $\partial(\pi_U + \pi_1)/\partial p_1 = 0$ and $\partial\pi_2/\partial p_2 = 0$ simultaneously yields

$$p_1 = \frac{\alpha(2\beta^2 - r^2 - \beta r) + 3\beta w_2 r}{4\beta^2 - r^2},$$

$$p_2 = \frac{\alpha(2\beta^2 - r^2 - \beta r) + 2\beta^2 w_2 + w_2 r^2}{4\beta^2 - r^2}.$$

Then, solving $\partial(\pi_U + \pi_1)/\partial w_2 = 0$ yields $w_2^{VI} = \frac{1}{2}\frac{\alpha(r^3+8\beta^3)}{\beta(r^2+8\beta^2)}$ in the first stage. The social welfare is $W^{VI} = \frac{\alpha(304\beta^5+48\beta^4 r+108\beta^3 r^2+16\beta^2 r^3+11\beta r^4-r^5)}{8\beta(r+\beta)(r^2+8\beta^2)^2}$. The output prices are

$$p_1^{VI} = \frac{\alpha(2\beta + r)(4\beta - r)}{2(r^2 + 8\beta^2)},$$

$$p_2^{VI} = \frac{\alpha(12\beta^3 - 4r\beta^2 + 2r^2\beta - r^3)}{2\beta(r^2 + 8\beta^2)}.$$

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[23] If the upstream firm is allowed to merge firm 1 and firm 2 simultaneously, then the merger results in a monopoly. Our major results still hold in this less interesting case.

3.3 Horizontal Integration

Suppose firm 1 and firm 2 merge together, then solving $\partial(\pi_1+\pi_2)/\partial p_1=0$ and $\partial(\pi_1+\pi_2)/\partial p_2=0$ simultaneously yields $p_i=\frac{1}{2}(w_i+\alpha),\ i,j=1,2,\ i\neq j$. Plugging p_1 and p_2 into π_U and solving $\partial\pi_U/\partial w_1=0$ and $\partial\pi_U/\partial w_2=0$ simultaneously yields $w_1^{HI}=w_2^{HI}=\frac{\alpha}{2}$, and $p_1^{HI}=p_2^{HI}=\frac{3\alpha}{4}$. The social welfare for this horizontal integration is $W^{HI}=\frac{7\alpha^2}{16(r+\beta)}$.

3.4 Welfare Ranking

The social welfare in different scenarios can be compared as follows:

$$W^N-W^{HI}=\frac{\alpha^2(12\beta-7r)}{16(r+\beta)(r-2\beta)^2}>0,$$

$$W^{VI}-W^N$$

$$=\frac{\alpha^2(\beta-r)(320\beta^6-192\beta^5r+128\beta^4r^2-64\beta^3r^3+10\beta^2r^4-14\beta r^5+r^6)}{8(r+\beta)(r-2\beta)^2\beta(r^2+8\beta^2)^2}>0.$$

Note that the assumption $\beta>r$ ensures the above ranking. Henceforth, the ranking of welfare is $W^{VI}>W^N>W^{HI}$. Our benchmark case reveals that vertical integration is welfare superior to the unintegrated case due to avoiding double marginalization, while horizontal integration is welfare inferior to the separated case due to a higher market concentration.

We can further compare the input prices and output prices. Vertical integration will result in a lower input price w_2^{VI} than the other two cases w_2^N and w_2^{HI}, which are both monopoly prices. The output prices comparison yields $p_2^{HI}>p_2^N>p_2^{VI}>p_1^{VI}$. Intuitively, a horizontal integration in the output market has the highest output price, because the output market becomes a monopolistic market. Vertical integration provides lower output prices due to the elimination of double marginalization and lower input prices.

4. Welfare Comparison with Digital Goods

Now we allow the emergence of a digital good. The profit function of the upstream firm becomes $\pi_U = w_1 Q_1 + w_2 Q_2 + p_e Q_e$, where the last term is the profit coming from the digital good. For simplicity, we assume $\beta = 1$ and $r = 1/2$[24]. Similar calculations with the benchmark case yield the equilibrium prices

$$w_1^N = w_2^N = \frac{\alpha(12\eta^4 + \eta^3 - 30\eta^2 + 18)}{2(12\eta^4 - 29\eta^2 + 18)},$$

$$p_1^N = p_2^N = \frac{\alpha(12\eta^4 + 5\eta^3 - 34\eta^2 - 6\eta + 24)}{2(12\eta^4 - 29\eta^2 + 18)},$$

$$p_e^N = \frac{\alpha(16\eta^4 - 4\eta^3 - 33\eta^2 + 4\eta + 18)}{2(12\eta^4 - 29\eta^2 + 18)}.$$

The above equilibrium prices are always positive, since $\eta < \beta$. The social welfare of the no-integration case becomes

$$W^N = \frac{\alpha^2(5796 - 6192\eta - 19428\eta^2 + 20616\eta^3 + 24505\eta^4 - 25848\eta^5 - 13820\eta^6)}{8(3 - 4\eta^2)(12\eta^4 - 297\eta^2 + 18)^2} + \frac{\alpha^2(14496\eta^7 + 2944\eta^8 - 307279)}{8(3 - 4\eta^2)(12\eta^4 - 297\eta^2 + 18)^2}.$$

Moreover, in the case of vertical integration, we have

$$w_2^{VI} = \frac{\alpha(64\eta^3 + 64\eta^2 - 66\eta - 65)}{4(1+\eta)(32\eta^2 - 33)},$$

$$p_1^{VI} = \frac{\alpha(32\eta^2 + 2\eta - 35)}{2(32\eta^2 - 33)},$$

$$p_2^{VI} = \frac{\alpha(64\eta^3 + 80\eta^2 - 66\eta - 83)}{4(1+\eta)(32\eta^2 - 33)},$$

$$p_e^{VI} = \frac{\alpha(36\eta^2 - 4\eta - 33)}{2(32\eta^2 - 33)},$$

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24 Generally, we have similar results for various values of β and r.

$$W^{VI} = \frac{\alpha^2(10623 - 1218\eta - 33136\eta^2 + 2368\eta^3 + 34288\eta^4 - 1152\eta^5 - 11776\eta^6)}{4(3-4\eta^2)(1+\eta)(33-32\eta^2)^2}.$$

Note that $\eta > 0$ ensures positive prices for p_e^{VI} and p_1^{VI}. For the horizontal integration, we have

$$w_1^{HI} = w_2^{HI} = \frac{\alpha(6+\eta^3)}{2(\eta^2+6)},$$

$$p_1^{HI} = p_2^{HI} = \frac{\alpha(9-\eta^3+2\eta^2-3\eta)}{2(\eta^2+6)},$$

$$p_e^{HI} = \frac{\alpha(6-\eta^2+2\eta)}{2(\eta^2+6)},$$

$$W^{HI} = \frac{\alpha^2(576(1-\eta)+180\eta^2-276\eta^3-7\eta^4-48\eta^5+4\eta^6)}{8(3-4\eta^2)(\eta^2+6)^2}.$$

To ensure positive prices and profits, η must belong the interval $(0, 0.8660)$. The welfare comparison and detailed calculations yield the following proposition.

Proposition 1. The welfare ranking will be changed to $W^{VI} > W^{HI} > W^N$ when $0.7071 < \eta < 0.8660$. That is, the no-integration case becomes the worst case when the physical product and the digital good are strong substitutes.

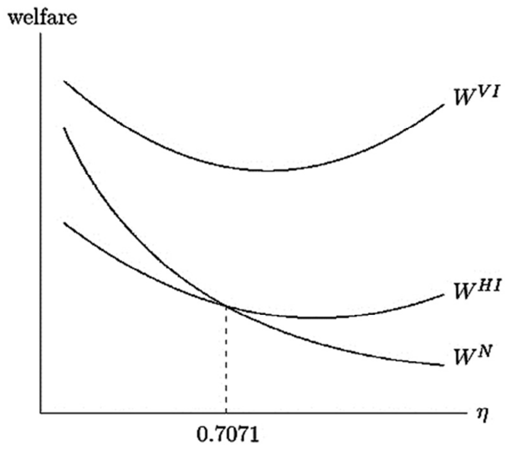

Figure 2. Welfare comparisons in different market structures.

Source: Organized by the authors.

Figure 2 depicts the welfare ranking around $\eta = 0.7071$. This result is a new finding in the related literature. Specifically, horizontal integration could be welfare superior to the no-integration case. If we consider the benefit from cost efficiency of mergers, namely that horizontal integration reduces average unit cost, then the welfare ranking is more likely to appear. The most important policy implication is that horizontal integration may be socially desirable in the case with digital goods. Intuitively, when η is large, the upstream firm has more ability to obtain profits via the digital version; given that η is large, the downstream physical market becomes a monopoly when downstream firms are merged. Therefore, the upstream firm will focus more on the profit from the digital version and thus decrease both the input prices and the output prices of the physical and digital goods (as shown in Proposition 2), which raises the welfare of consumers. In fact, horizontal integration may result in lower physical and digital prices. This is because the profit for the upstream firm comes from two parts, from selling the input and from selling the digital good. In the case of large η, horizontal integration results in lower sales on the physical good, and consequently also decreases the profit from selling the input. Therefore, the upstream firm has an incentive to set a lower price $p_1^{HI} < p_1^N$ to enlarge the sales of the physical good and consequently to increase the profit from selling the input. Notably, this result is robust for various values of r. For instance, if r is replaced by $1/4$ $(3/4)$, then $W^{HI} > W^N$ whenever $\eta > 0.5(0.866)$. Proposition 1 also shows that vertical integration is socially superior to the other two scenarios. However, this framework does not consider the incentive to foreclose competition such as Chen (2001). In our setting, since there exist no potential entrants, vertical integration is then always welfare enhancing, due to the elimination of double marginalization, with no regards to the introduction of a digital good or not.

We can further discuss the effects of η on the degree of double marginalization and market power. Since there are zero production costs in our setting, the level of equilibrium prices of the physical good can be seen as a measure of market power. By detailed calculations, we obtain $\partial p_1^N / \partial \eta < 0$, $\partial p_1^{VI} / \partial \eta < 0$, and $\partial p_1^{NI} / \partial \eta < 0$, meaning that an increase in η leads to a decrease in the degree of market power, from which it is intuitively clear that the more substitution there is between the physical good and digital good, the less market power firms will have. We can also measure the degree of double marginalization by the price difference $p_1^N - p_1^{VI}$ and $p_1^{NI} - p_1^{VI}$, since the case of vertical integration can be seen as zero double marginalization. By detailed calculations, we have $\partial(p_1^N - p_1^{VI})/\partial\eta < 0$ and $\partial(p_1^{NI} - p_1^{VI})/\partial\eta < 0$, meaning that a large η is associated with a low degree of double marginalization.

By comparing prices of the digital good and the physical good among different cases, we obtain the following proposition.

Proposition 2. The comparisons for the prices of the digital and physical good are as follows. (i) $p_e^N > p_e^{VI}$ iff $0 < \eta < 0.8094$. (ii) $p_e^N > p_e^{HI}$ iff $0.7071 < \eta < 0.8660$. (iii) $p_e^{HI} > p_e^{VI}$ iff $\eta < 0.7859$. (iv) $p_1^N > p_1^{VI}$ and $p_1^{HI} > p_1^{VI}$. (v) $p_1^N > p_1^{HI}$ iff $0.7071 < \eta < 0.8660$.

The above result implies various ranking between p_e^N, p_e^{VI}, and p_e^{HI}. For the range of positive η, there are four different rankings among them:

(i) $p_e^{HI} > p_e^N > p_e^{VI}$ if $0 < \eta < 0.7071$;

(ii) $p_e^N > p_e^{HI} > p_e^{VI}$ if $0.7071 < \eta < 0.7859$;

(iii) $p_e^N > p_e^{VI} > p_e^{HI}$ if $0.7859 < \eta < 0.8094$;

(iv) $p_e^{VI} > p_e^N > p_e^{HI}$ if $0.8094 < \eta < 0.8660$.

Intuitively, when η is relatively small, the effect of double marginalization is more likely to be reduced, and therefore p_e^{VI} is the lowest one. On the other hand, when η is relatively large, the upstream firm is more likely to set the lowest price in the horizontal case. Proposition 2 also reveals that vertical integration provides the lowest physical price, and horizontal integration may surprisingly result in a lower physical price than that of no integration, because of the emergence of the digital good.

Proposition 3. *In general, the electronic version is cheaper than the physical version under different market structures. Specifically,* (i) $p_e^N < p_1^N < p_2^N$*;* (ii) $p_e^{HI} < p_1^{HI} < p_2^{HI}$*;* (iii) $p_e^{VI} < p_2^{VI}$, $p_e^{VI} < p_1^{VI}$, *if* $\eta < \frac{1}{2}$ *and* $p_e^{VI} < \frac{p_1^{VI} q_1^{VI} + p_2^{VI} q_2^{VI}}{q_1^{VI} + q_2^{VI}}$ *(the weighted average price).*

A numerical illustration is given for $\eta = 0.8$ and $\alpha = 5$. The equilibrium prices are $p_1^N = 0.56$, $p_e^N = 0.53$, $p_1^{VI} = 0.52$, $p_2^{VI} = 0.57$, $p_e^{VI} = 0.53$, $p_1^{HI} = 0.55$, $p_e^{HI} = 0.52$, which reveals that the electronic versions are all cheaper than the physical versions, and the output price (p_1^{HI} and p_e^{HI}) for the horizontal integration case will be less than that of the non-integration case. Moreover, the social welfare is $SW^N = 0.45$, and $SW^{VI} = 0.47$, and $SW^{HI} = 0.46$.

We also find that the input price comparison may be affected by the parameter η. Specifically,

$$w_2^{HI} > w_2^{NI} \quad \text{iff} \quad 0.8660 > \eta > \frac{\sqrt{2}}{2} \doteqdot 0.707,$$

$$w_2^{HI} > w_2^{VI} \quad \text{iff} \quad \eta < 0.345 \quad \text{or} \quad 0.8660 > \eta > 0.775,$$

$$w_2^{NI} > w_2^{VI} \quad \text{iff} \quad \eta < 0.567.$$

Intuitively, for example, when η is relatively large, meaning that the substitution effect between the physical good and the digital good is large, w_2^{HI} is the highest, because of the concentration of the output market. The input price w_2^{VI} is higher than w_2^{NI}, because the upstream firm tries to raise its rival's cost for the benefit of the digital good.

5. Extensions

5.1 Welfare Comparison with Electronic Version of Product and Bargaining Power

So far, we implicitly assume that the upstream firm dominates the downstream firms. That is, the upstream first decides the input prices, then the downstream firms set their product prices. For a more realistic situation, both sides may have some bargaining power in determining the input prices. For simplicity, we assume $k \in [0,1]$ is the bargaining power of the upstream firm, while $1 - k$ is the bargaining power of the downstream firm.

The objective functions become $k\pi_U + (1-k)\pi_i,\ i = 1,2$ for solving input prices w_1 and w_2[25]. Other settings in the benchmark case are unchanged.

We start to analyze the benchmark case with bargaining power. In the case without integration, the input prices can be obtained by solving $\partial(k\pi_U + (1-k)\pi_1)/\partial w_1 = 0$ and $\partial(k\pi_U + (1-k)\pi_2)/\partial w_2 = 0$:

$$w_1^N = w_2^N = \frac{\alpha(4\beta^2(2k-1) + r^2(2-3k))}{2(2\beta^2(3k-1) + r^2(1-2k))}.$$

The higher the bargaining power of the upstream firm, the higher the input prices, because

$$\frac{\partial w_1^N}{\partial k} = \frac{\alpha(2\beta + r)(2\beta - r)(2\beta^2 - r^2)}{2\beta^2(3k-1) + r^2(1-2k)} > 0.$$

In the case with vertical integration between the upstream firm and the first downstream firm, the input prices can be obtained by solving $\partial(k(\pi_U + \pi_1) + (1-k)\pi_2)/\partial w_2 = 0$ to yield

$$w_2^{VI} = \frac{\alpha(2\beta + r)(4\beta^2(2k-1) + 2\beta r(2-3k) + kr^2)}{2\beta(4\beta^2(3k-1) + r^2(4-3k))}.$$

The comparative statics shows that

$$\frac{\partial w_1^{VI}}{\partial k} = \frac{2\alpha(\beta - r)(2\beta - r)(2\beta + r)(2\beta^2 + r^2)}{4\beta^2(3k-1) + r^2(4-3k)} > 0,$$

meaning that the higher the bargaining power of the upstream firm, the higher the input price w_2.

In the case of horizontal integration for downstream firms, the input prices can be obtained by solving $\partial(k\pi_U + (1-k)(\pi_1 + \pi_2))/\partial w_1 = 0$ and $\partial(k\pi_U + (1-k)(\pi_1 + \pi_2))/\partial w_2 = 0$ simultaneously. We then have $w_1^{HI} = w_2^{HI} = \frac{\alpha(2k-1)}{3k-1}$. Since $\frac{\partial w_2^{HI}}{\partial k} = \frac{\alpha}{(3k-1)^2} > 0$, we have that the higher the bargaining power of the upstream firm is, the higher the input price w_2 will be. The welfare comparison for the benchmark case with bargaining power leads to the following proposition.

[25] An alternative setting of the objective function is $\pi_U^k \cdot \pi_i^{1-k}$, as generalized Nash bargaining. However, the calculation is unmanageable.

Proposition 4. In the case with bargaining power, (i) $W^{VI} > W^{N} \geq W^{HI}$ when k is not close to $1/2$ (we use $k = 0$). (ii) $W^{N} > W^{VI} > W^{HI}$ when k is close to $1/2$ and r is not close to β. (iii) $W^{N} > W^{HI} > W^{VI}$ when k is close to $1/2$ and r is close to β.

Proposition 4 reveals that the welfare ranking under the benchmark case will reverse when the upstream and downstream firms have nearly equal bargaining power. Specifically, vertical integration could be the worst case, which is seldom discussed in the literature of double marginalization. Intuitively, when k is close to $1/2$, the effect of double marginalization is low. The social benefit from vertical integration due to the elimination of double marginalization is low. Thus, vertical integration could be the worst case under some conditions.

We now extend to the case with an electronic version of the product and bargaining power with $\beta = 1$ and $r = 1/2$. We have the following two propositions for the case $k = 1/2$. It is worth mentioning that the first-order conditions in the case of no integration are $\partial(\pi_1 + \pi_U)/\partial p_1 = 0$, $\partial(\pi_2 + \pi_U)/\partial p_2 = 0$, $\partial(\pi_1 + \pi_U)/\partial w_1 = 0$, $\partial(\pi_2 + \pi_U)/\partial w_2 = 0$, and $\partial\pi_U/\partial p_e = 0$, while the first-order conditions in the case of vertical integration are $\partial(\pi_1 + \pi_U)/\partial p_1 = 0$, $\partial(\pi_2 + (\pi_U + \pi_1))/\partial p_2 = 0$, $\partial((\pi_U + \pi_1) + \pi_2)/\partial w_2 = 0$, and $\partial(\pi_U + \pi_1)/\partial p_e = 0$. Both cases have similar but still different first-order conditions.

Proposition 5. When $k = 1/2$, (i) $W^{N} > W^{VI}$ iff $0 \leq \eta < 0.846$. (ii) $W^{N} > W^{HI}$ iff $0 \leq \eta < 0.851$. (iii) $W^{HI} > W^{VI}$ iff $0.106 < \eta < 0.318$. Therefore, $W^{VI} > W^{N} > W^{VI}$ when k is not close to $1/2$. Moreover, $W^{N} > W^{HI} > W^{VI}$ when $k = 1/2$ and $0.106 < \eta < 0.318$.

With the bargaining power, we have a proposition similar to that in Proposition 2.

Proposition 6. *When $k = 1/2$, the comparison for prices of the electronic product are as follows:* (i) $p_e^{N} > p_e^{HI} > p_e^{VI}$ *if* $\eta < 0$, (ii) $p_e^{VI} > p_e^{HI} > p_e^{N}$ *if* $0 < \eta < 0.760$, (iii) $p_e^{HI} > p_e^{VI} > p_e^{N}$ *if* $\eta > 0.760$. *Moreover, the electronic version is generally cheaper* *than the physical version when* η *is not low:* (i) $p_e^N < p_1^N (= p_2^N)$ *iff* $\eta > 0.333$, (ii) $p_e^{HI} < p_1^{HI} (= p_2^{HI})$ *iff* $\eta > 0$, (iii) $p_e^{VI} < p_1^{VI}$ *iff* $\eta > 0.5$, $p_e^{VI} < p_2^{VI}$ *iff* $\eta \geq 0$, *and* $p_e^{VI} < \frac{p_1^{VI} \cdot q_1^{VI} + p_2^{VI} \cdot q_2^{VI}}{q_1^{VI} + q_2^{VI}}$ *iff* $\eta > 0.238$.

Due to the complexity of calculations, we provide the influence of bargaining power (k) on the equilibrium prices and social welfare by numerical illustrations summarized in Table 1. Given $\eta = 0.7$, we find that when k increases, all prices rise. Intuitively, when the bargaining power of the upstream firm increases, it can charge higher input prices, which induces higher output prices (p_1 and p_2), and consequently, the price of the digital good (p_e) also increases due to the substitution effect. Moreover, social welfare is reduced when k increases, due to the above price effects. Therefore, the increase of the bargaining power for the upstream firm will induce a more serious double marginalization problem. That is, prices increase and welfare decreases.

Table 1. Influence of k on prices and welfare ($\eta = 0.7$).

Equilibrium	$k = 0.4$	$k = 0.5$	$k = 0.6$
$p_1^N (= p_2^N)$	0.331α	0.446α	0.5α
p_e^N	0.27α	0.391α	0.448α
w^N	$0.598\alpha^2$	0.54α	$0.506\alpha^2$
$p_1^{HI} (= p_2^{HI})$	0.469α	0.513α	0.539α
p_e^{HI}	0.415α	0.461α	0.488α
w^{HI}	$0.525\alpha^2$	$0.497\alpha^2$	$0.48\alpha^2$
p_1^{VI}	0.446α	0.477α	0.493α
p_2^{VI}	0.459α	0.511α	0.538α
p_e^{VI}	0.424α	0.468α	0.49α
w^{VI}	$0.536\alpha^2$	$0.51\alpha^2$	$0.495\alpha^2$

Source: Organized by the authors.

This result of various welfare rankings shown in Proposition 5 and comparison for prices in Proposition 6 may still be satisfied when k is not $1/2$. Proposition 5 indicates that the efficiency loss from double marginalization is reduced, due to the equal weight of bargaining power between the upstream and the downstream firms. Table 1 presents the influence of different k on Propositions 5 and 6. For instance, considering that $k = 0.6$, calculations lead to three possible welfare rankings as follows:

(i) $W^{VI} > W^{HI} > W^{N}$ if $\eta > 0.844$,

(ii) $W^{VI} > W^{N} > W^{HI}$ if $\eta < 0.830$,

(iii) $W^{N} > W^{VI} > W^{HI}$ if $0.830 < \eta < 0.844$.

Table 2. Influence of different bargaining power.

Conditions	$k = 0.4$	$k = 0.5$	$k = 0.6$	$k = 0.7$
$W^{N} > W^{VI}$	$\eta < 0.853$	$\eta < 0.846$	$\eta < 0.830$	N.A.
$W^{N} > W^{HI}$	$0.157 < \eta < 0.856$	$\eta < 0.851$	$\eta < 0.844$	$\eta < 0.829$
$W^{HI} > W^{VI}$	$\eta < 0.613$	$0.106 < \eta < 0.318$	N.A.	N.A.
$p_e^{VI} > p_e^{HI} > p_e^{N}$	$0.171 < \eta < 0.739$	$\eta < 0.760$	$\eta < 0.770$	$\eta < 0.776$
$p_e^{HI} > p_e^{VI} > p_e^{N}$	$\eta > 0.739$	$\eta > 0.760$	$\eta > 0.770$	$\eta > 0.776$
$p_e^{VI} > p_e^{N} > p_e^{HI}$	$\eta < 0.717$	N.A.	N.A.	N.A.
$p_1^{N} > p_e^{N}$	$\eta > 0.512$	$\eta > 0.333$	$\forall\, \eta$	$\forall\, \eta$
$p_1^{VI} > p_e^{VI}$	$\eta > 0.5$	$\eta > 0.5$	$\eta > 0.5$	$\eta > 0.5$
$p_1^{HI} > p_e^{HI}$	$\eta > 0.345$	$\forall\, \eta$	$\forall\, \eta$	$\forall\, \eta$

∗ N.A. indicates that the condition is never satisfied for all η.

Source: Organized by the authors.

This finding is different from Proposition 5 that W^{N} could be welfare superior to the other cases, because nearly equal bargaining power between firms reduces the efficiency losses from double marginalization, and thus the anticompetitive effect dominates the welfare comparison. Moreover, Table 2 also shows that the price of the electronic product is lower than that of the physical product when η is not small for variance values of k, which is consistent with Proposition 6(3).

5.2 An Extension with an Outside Digital Product Provider

Now we extend to a new case such that the electronic version of the product is provided by an independent outside competitor. Therefore, the profit of the upstream firm becomes $\pi_U = w_1 Q_1 + w_2 Q_2$, and the profit of the outside competitor is $\pi_e = p_e Q_e$. For simplicity, we assume $\beta = 1$ and $r = 1/2$.[26] Detailed calculations similar to those in the benchmark case yield the equilibrium prices:

$$w_1^N = w_2^N = \frac{1}{4}\frac{(4\eta^2 + 3\eta - 6)}{2\eta^2 - 3},$$

$$p_1^N = p_2^N = \frac{1}{2}\frac{(5\eta^2 - 6)(4\eta^2 + 3\eta - 6)}{(8\eta^2 - 9)(2\eta^2 - 3)},$$

$$p_e^N = -\frac{1}{2}\frac{(8\eta^5 - 20\eta^3 - 26\eta^4 + 54\eta^2 + 12\eta - 27)}{(8\eta^2 - 9)(2\eta^2 - 3)}.$$

Moreover, in the case of vertical integration, we have

$$w_2^{VI} = \frac{1}{4}\frac{(-5 + 6\eta^2)(4\eta^2 + 3\eta - 6)(36\eta^4 - 74\eta^2 + 39)}{504\eta^8 - 2106\eta^6 + 3284\eta^4 - 2277\eta^2 + 594},$$

$$p_1^{VI} = \frac{1}{4}\frac{(4\eta^2 + 3\eta - 6)(48\eta^4 - 89\eta^2 + 42)(6\eta^2 - 5)}{504\eta^8 - 2106\eta^6 + 3284\eta^4 - 2277\eta^2 + 594},$$

$$p_2^{VI} = \frac{3}{4}\frac{(\eta - 1)(\eta + 1)(96\eta^4 - 174\eta^2 + 83)(4\eta^2 + 3\eta - 6)}{504\eta^8 - 2106\eta^6 + 3284\eta^4 - 2277\eta^2 + 594},$$

$$p_e^{VI} = -\frac{1}{4}\frac{(576\eta^9 - 1584\eta^8 - 2352\eta^7 + 5796\eta^6 + 3632\eta^5 - 8036\eta^4)}{504\eta^8 - 2106\eta^6 + 3284\eta^4 - 2277\eta^2 + 594} + \frac{(-2524\eta^3 + 5013\eta^2 + 666\eta - 1188)}{504\eta^8 - 2106\eta^6 + 3284\eta^4 - 2277\eta^2 + 594}.$$

For the horizontal integration, we have

$$w_1^{HI} = w_2^{HI} = \frac{1}{4}\frac{(4\eta^2 + 3\eta - 6)}{2\eta^2 - 3},$$

$$p_1^{HI} = p_2^{HI} = \frac{1}{8}\frac{(2\eta + 3)(2\eta - 3)(4\eta^2 + 3\eta - 6)}{(2\eta^2 - 3)(\eta^2 - 3)},$$

[26] Generally, we have similar results for various values of β and r.

$$p_e^{HI} = \frac{1}{4}\frac{(8\eta^4 + 4\eta^3 - 27\eta^2 - 6\eta + 18)}{(2\eta^2 - 3)(\eta^2 - 3)}.$$

Similarly, we require $0 \leq \eta < 0.839$ to ensure all the above prices are positive. Then we have welfare comparison and price comparison as the following propositions by detailed calculation.

Proposition 7. In the case with an outside competitor, the welfare comparison becomes $W^{VI} > W^N$ and $W^{VI} > W^{HI}$, consistent with the benchmark case; however, $W^{HI} > W^N$ iff $0.8397 > \eta > 0.7071$.

Proposition 7 reveals the possibility of the welfare ranking $W^{VI} > W^{HI} > W^N$ when η is close to 1.[27]

Proposition 8. *For the price of the electronic version of the product, we have* (i) $p_e^N - p_e^{VI} > 0$ *iff* $\eta \geq 0$, (ii) $p_e^N - p_e^{HI} > 0$ *iff* $0.7071 < \eta < 0.839$, (iii) $p_e^{HI} - p_e^{VI} > 0$ *iff* $0 \leq \eta < 0.839$. *In addition, the electronic version is generally cheaper than the physical version under different market structures. Specifically,* (iv) $p_e^N < p_1^N < p_2^N$, (v) $p_e^{HI} < p_1^{HI} < p_2^{HI}$, (vi) $p_e^{VI} < p_2^{VI}$, $p_e^{VI} < p_1^{VI}$, *if* $\eta \geq 0$ *and* $p_e^{VI} < \frac{p_1^{VI}q_1^{VI} + p_2^{VI}q_2^{VI}}{q_1^{VI} + q_2^{VI}}$ *(the weighted average price).*

5.3 Rapid Growth in the Digital Market

Consider different values of the reservation price for the digital product such that $p_e = \alpha_e - \beta Q_e - \eta(Q_1 + Q_2)$, where α_e represents the size of the market. A large α_e means a larger market. This setting allows us to analyze various market scales of digital

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[27] Intuitively, given that η is large, the substitutability between the digital and physical goods is high. Horizontal integration creates an environment where the integrated downstream firm should be the only firm that directly competes against the outside digital product provider, resulting in lower digital and physical prices. Therefore, the social welfare under horizontal integration is higher than that of no integration.

goods, which may be related to the increasing concern on the definition of "market" in the fast growing digital market. We have the following proposition.

Proposition 9. *When the digital market is large (α_e is large), both vertical integration and horizontal integration may not be socially desirable.*

5.4 Implications for Antitrust Policy

Our study provides a framework with digital-physical mashups to examine the welfare comparison between various market structures. In this subsection, we attempt to provide implications for merger policy in digital industries. First, Proposition 1 and Figure 2 reveal a possibility that a horizontal merger between downstream firms can be socially beneficial when the substitutability between the physical and digital version of product is large. The antitrust authorities may take into account the situation of substitutability between physical and digital versions as one of the key factors for merger policies in different cases. Considering media industries as an example, the rise of the Internet allows content providers to distribute their products through direct channels, which can change the original bottleneck of downstream cable TV. Henceforth, the antitrust concern of mergers in the downstream cable industry is reduced and more merger cases have appeared in recent years.

Second, another policy implication is associated with vertical integration between the upstream firm and a part of downstream firms. The analytical result reveals that this type of vertical merger is welfare improving due to the effect of reducing double marginalization. A policy implication is to examine whether the double marginalization is reduced significantly from vertical mergers with the role of directly-channelled electronic versions of products.

6. Conclusion

This study investigates the welfare ranking of vertical integration and horizontal integration with a physical product and its directly-channelled electronic version competing in an upstream-downstream structure. Without cost differentiation and information asymmetry, traditional wisdom says that vertical integration is superior to non-integration and horizontal integration. Introducing an electronic version of the product or a direct channel to consumers may result in a reversed welfare ranking compared to that in the traditional wisdom. The intuition of our results is that the substitutability of the electronic version for the physical version reduces the degree of double marginalization. Our policy implication suggests horizontal merger could be socially desirable when digital goods are included. A natural extension of our study is to allow for vertical differentiation between the electronic version and the physical version.

In a larger sense, the digital version of a product may be more powerful than its physical version in some industries, even in the gym equipment industry, where a recent successful player, Pelonton, provided direct-channel sales and the associated on-demand classes. Amazon Kindle, another well-known example, was developed for providing information and entertainment, but now has been extended to health service and can support smart speakers to have immediate feedback and voice purchases, which is infeasible for a physical channel. Broader discussion on how merger policy should adapt to the digital era considering big data, start-ups, and market boundaries awaits further studies.

Appendix

Proof of Proposition 1

Proof. The welfare ranking can be derived by detailed calculations such that

$$W^{VI} - W^{N} = \alpha^2 \Phi_1(\eta) > 0,$$
$$W^{VI} - W^{HI} = \alpha^2 \Phi_2(\eta) > 0,$$
$$W^{N} - W^{HI} = \alpha^2 \Phi_3(\eta) > 0, \quad \text{when } \eta > 0.7071,$$

where Φ_i, $i = 1,2,3$, are functions in η. □

Proof of Proposition 2

Proof. By calculations, $p_e^N - p_e^{VI} = \alpha \cdot \Phi_4(\eta) > 0$ if $\eta < 0.8094$. Other cases are obtained by similar computations. □

Proof of Proposition 3

Proof. (i) $p_e^N - p_1^N = \frac{-\alpha(1-\eta)(4\eta^3-5\eta^2-47+6)}{2(12\eta^4-29\eta^2+18)} < 0$. (ii) $p_e^{HI} - p_1^{HI} = \frac{-\alpha(1-\eta)(\eta^2-2\eta+3)}{2(\eta^2+6)} < 0$. (iii) $p_e^{VI} - p_1^{VI} = \frac{\alpha(1-2\eta)(1-\eta)}{33-32\eta^2} < 0$ if $\eta < \frac{1}{2}$, and $p_e^{VI} - p_2^{VI} = \frac{-\alpha(8\eta^3-16\eta^2-8\eta+17)}{4(1+\eta)(33-32\eta^2)} < 0$. Finally, the comparison between p_e^{VI} and the weighted average price can be calculated in detail. □

Proof of Proposition 4

Proof. The comparison between W^{VI} and W^N yields

$$W^{VI} - W^{N}|_{k=0} = \frac{\alpha^2(4r+3\beta)}{8(r+\beta)^2} > 0,$$

$$W^{VI} - W^{N}|_{k=1/2} = \frac{\alpha^2 r(5r^3 - 14\beta r^2 - 10r\beta^2 - 8\beta^3)}{16\beta^2(5r^2+4\beta^2)(r+\beta)} < 0,$$

which shows W^{VI} (W^{N}) is welfare superior to other structures when k is (not) close to $1/2$. Moreover, $W^{N} > (=) W^{HI}$ is valid for all $k > (=) 0$ by detailed calculations. Finally, the welfare comparisons between vertical integration and horizontal integration are

$$W^{VI} - W^{HI}|_{k=0} = \frac{\alpha^2(4r+3\beta)}{8(r+\beta)^2} > 0,$$

$$W^{VI} - W^{HI}|_{k=1/2} = \frac{\alpha^2 r(4\beta-3r)(\beta-r)}{8(5r^2+4\beta^2)(r+\beta)\beta} < 0, \quad \text{if } r > \frac{3}{4}\beta,$$

which implies horizontal integration may be welfare superior to vertical integration when k is close to $1/2$ and r is close to β. □

Proof of Proposition 5

Proof. When $k = 1/2$, the equilibrium prices are calculated to become

$$W_1^N = W_1^N = \frac{\alpha(6+42\eta-34\eta^2-79\eta^3+44\eta^4+36\eta^5-16\eta^6)}{2(48-113\eta^2+80\eta^4-16\eta^6)},$$

$$p_1^N = p_2^N = \frac{\alpha(36+12\eta-90\eta^2-23\eta^3+68\eta^4+12\eta^5-16\eta^6)}{2(48-113\eta^2+80\eta^4-16\eta^6)},$$

$$p_e^N = \frac{\alpha(48-36\eta-77\eta^2+68\eta^3+12\eta^4-32\eta^5+16\eta^6)}{2(48-113\eta^2+80\eta^4-16\eta^6)},$$

$$W_2^{VI} = \frac{\alpha(25+42\eta-16\eta^2-32\eta^3)}{4(\eta+1)(21-16\eta^2)},$$

$$p_1^{VI} = \frac{\alpha(19+2\eta-16\eta^2)}{2(21-16\eta^2)},$$

$$p_2^{VI} = \frac{\alpha(43+42\eta-32\eta^2(1+\eta))}{2(\eta+1)(21-16\eta^2)},$$

$$p_e^{VI} = \frac{\alpha(2\eta+3)(7-6\eta)}{2(21-16\eta^2)},$$

$$W_1^{HI} = W_2^{HI} = \frac{\alpha\eta(\eta+3)(\eta+1)}{2(5\eta^2+3)},$$

$$p_1^{HI} = p_2^{HI} = \frac{\alpha(3+6\eta^2-\eta^3)}{2(5\eta^2+3)},$$

$$p_e^{HI} = \frac{\alpha(3 - 2\eta + 7\eta^2)}{2(5\eta^2 + 3)}.$$

Notably, η must belong to the range $[0, 0.866]$ to ensure positive prices and profits. After further detailed calculations, we have the following welfare comparisons

$$W^N > W^{VI} \quad \text{iff} \quad 0 \leq \eta < 0.846,$$

$$W^N > W^{HI} \quad \text{iff} \quad 0 \leq \eta < 0.851,$$

$$W^{HI} > W^{VI} \quad \text{iff} \quad 0.106 < \eta < 0.318. \qquad \square$$

Proof of Proposition 6

Proof. By detailed calculations, we have

$$p_e^N = \frac{\alpha(16\eta^6 - 32\eta^5 + 12\eta^4 + 68\eta^3 - 77\eta^2 - 36\eta + 48)}{(48 + 80\eta^4 - 16\eta^6 - 113\eta^2)},$$

$$p_e^{VI} = \frac{\alpha(2\eta + 3)(6\eta - 7)}{2(16\eta^2 - 21)},$$

$$p_e^{HI} = \frac{\alpha(7\eta^2 - 2\eta + 3)}{2(5\eta^2 + 3)},$$

which proved the first part of this proposition. For the second part,

$$p_1^N - p_e^N = \frac{\alpha(1 - \eta)(32\eta^5 - 12\eta^4 - 68\eta^3 + 23\eta^2 + 36\eta - 12)}{2(48 + 80\eta^4 - 16\eta^6 - 113\eta^2)} > 0 \quad \text{iff} \quad \eta > 0.333,$$

$$p_1^{HI} - p_e^{HI} = \frac{\alpha\eta(\eta + 2)(1 - \eta)}{2(5\eta^2 + 3)} > 0 \quad \text{iff} \quad \eta > 0,$$

$$p_1^{VI} - p_e^{VI} = \frac{\alpha(2\eta - 1)(1 - \eta)}{21 - 16\eta^2} > 0 \quad \text{iff} \quad \eta > 0.5,$$

$$p_2^{VI} - p_e^{VI} = \frac{\alpha(1 + 8\eta - 8\eta^3)}{4(\eta + 1)(21 - 16\eta^2)} > 0 \quad \text{iff} \quad \eta \geq 0,$$

and

$$\frac{p_1^{VI} \cdot q_1^{VI} + p_2^{VI} \cdot q_2^{VI}}{q_1^{VI} + q_2^{VI}} - p_e^{VI}$$

$$= \frac{2\alpha(1 - \eta)^2(66\eta + 112\eta^2 - 56\eta^3 - 80\eta^4 - 21)}{(21 - 16\eta^2)(80\eta^4 + 87 - 168\eta^2)} > 0, \quad \text{iff} \quad \eta > 0.238.$$

Proof of Proposition 7

Proof. By calculations, we obtain W^N, W^{VI}, and W^{HI} which are all complicated functions of a single parameter η, and their relationships are $W^{VI} > W^N$, $W^{VI} > W^{HI}$, and $W^N > W^{HI}$ iff $0 \leq \eta < 0.7071$. □

Proof of Proposition 8

Proof.

(i) By calculations, $p_e^N - p_e^{VI} = \alpha \cdot \Phi_5(\eta) > 0$ iff $\eta \geq 0$. Other cases of (ii) and (iii) can also be applied. (iv) $p_e^N - p_1^N = -\frac{1}{2}\frac{(\eta-1)(8\eta^4+2\eta^3-3\eta^2-3\eta-9)}{(8\eta^2-9)(2\eta^2-3)} < 0$, (v) $p_e^{HI} - p_1^{HI} = -\frac{1}{8}\frac{(4\eta^3-6\eta^2-15\eta+18)}{(2\eta^2-3)(\eta^2-3)} < 0$, (vi)

$p_e^{VI} - p_1^{VI} = -\frac{1}{4}\frac{(\eta-1)\left(576\eta^8+144\eta^7-1344\eta^6-372\eta^5+938\eta^4+334\eta^3-99\eta^2-108\eta-72\right)}{(504\eta^8-2106\eta^6+3284\eta^4-2277\eta^2+594)} < 0$, if $\eta \geq 0$, and

$p_e^{VI} - p_2^{VI} = -\frac{1}{4}\frac{(576\eta^9-432\eta^8-1488\eta^7+828\eta^6+1202\eta^5-92\eta^4-211\eta^3-609\eta^2-81\eta+306)}{(504\eta^8-2106\eta^6+3284\eta^4-2277\eta^2+594)} < 0.$

Finally, the comparison between p_e^{VI} and the weighted average price can be calculated in detail. □

Proof of Proposition 9

Proof. After some calculations, we have

$W^N - W^{VI} = \left(F_1(\eta) - F_2(\eta)\alpha_e - F_3(\eta)\right) \cdot (\eta_e\alpha_e - 1)$, where $F_1(\eta) > 0$ and $F_2(\eta) > 0$,

where F_1, F_2, and F_3 are functions of η.

$W^N - W^{HI} = \left(F_4(\eta) - F_5(\eta)\alpha_e - F_6(\eta)\right) \cdot (\eta_e\alpha_e - 1)$, where $F_4(\eta)$

> 0 and $F_5(\eta) > 0$,

where F_4, F_5, and F_6 are functions of η. When α_e is large, W^{VI} and W^{HI} may both be larger than W^N.

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直接通路數位財貨的
垂直及水平結合之福利比較

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賴孚權

摘要

本文分析直接通路之數位版產品的出現如何影響不同結合程度（不結合、垂直結合、水平結合）下，價格及福利的排序變化。我們考慮一種數位與實體混搭的架構，其中一家上游廠商將一種產品賣給下游兩家相互競爭的經銷商，同時也將該產品的數位版透過直接通路銷售給消費者。直接與間接通路的競爭，使得消費者得以從經銷商購買實體產品或直接通路購買該產品的數位版。我們發現，傳統的福利排序有可能會反轉。本研究最重要的政策意涵是當數位版和實體版財貨之間有強烈替代性時，水平結合是一種福利增進，因為數位化的出現降低了雙重邊際化及市場集中的效果，同時也降低兩種版本財貨的價格。考慮相對的談判力，垂直結合可能是最糟糕的。此外，我們也討論了有一家外在的廠商提供數位版財貨時的情況，在所有討論的情境中，數位版財貨的價格一般來說低於實體版財貨的價格。兩個反托拉斯政策意涵如下：第一、由餘下游廠商間的水平結合有可能是福利增進，所以公平交易委員會審查合併案件時，宜將實體與數位產品間的替代程度納入考量。第二、當上游與部分下游廠商間垂直結合時，必須檢視其雙重邊際化的降低程度是否大到足以使該結合達到社會福利增進。

關鍵詞：垂直結合、水平結合、數位版財貨、社會福利、雙重邊際化

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